AUSTRALIAN NATIONAL UNIVERSITY
Department of Engineering

ENGN6612/4612 Digital Signal Processing and Control
Problem Set #8 Filter Structures

Q1

A digital filter defined by the transfer function:-

(a) $H(z) = \frac{1}{3}\left\{1 + z^{-1} + z^{-2}\right\}$

(b) $H(z) = \frac{2}{1 - 3z^{-1} + z^{-2}}$

For each $H(z)$:

- Identify the filter type (FIR or IIR).
- Find the difference equation.
- Draw the block diagram representation of the filter in Direct-Form I.

Q2

A digital filter defined by the transfer function:-

$$H(z) = \frac{1}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

Draw the block diagram representation of the filter in

- Direct-Form I.
- Cascade Form.
- Parallel Form.

Q3

A digital filter defined by the transfer function:-

(a) $H(z) = \frac{1 - \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$

(b) $H(z) = \frac{1 - 6z^{-1} + 8z^{-2}}{1 - \frac{2}{3}z^{-1} + \frac{1}{9}z^{-2}}$

For each $H(z)$, draw the block diagram representation of the filter in Direct-Form II.

Q4

A digital filter is shown in the block diagram shown below:

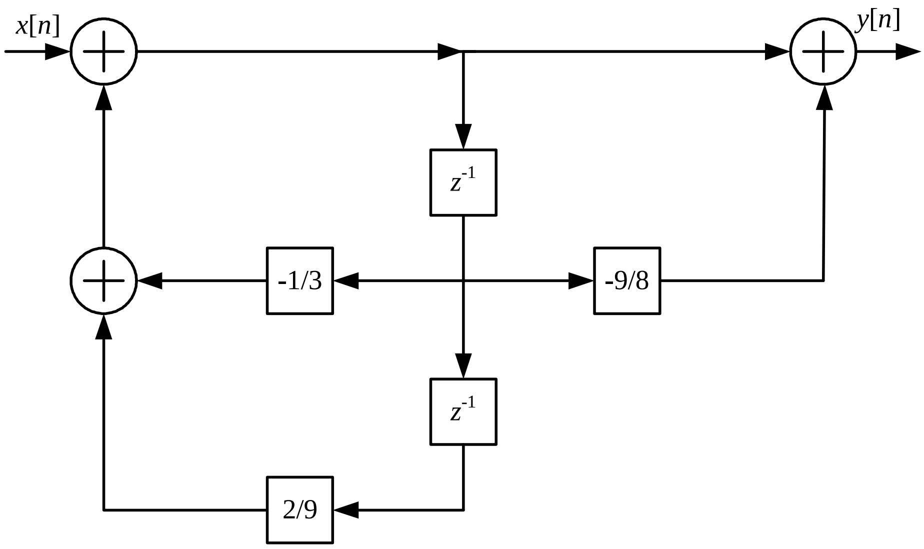

Figure 1: Question 4.

For the given filter, find the difference equation. `(challenge problem)`

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ENGN6612/4612 Digital Signal Processing and Control
Problem Set #8 Solution

Q1

(a) Complete Solution

Given that

$$H(z) = \frac{1}{3}\left\{1+z^{-1}+z^{-2}\right\}$$

This is a FIR filter (3-point moving-average FIR filter).

Re-writing the transfer function, we have

$$\begin{aligned} \frac{Y(z)}{X(z)} &= \frac{1}{3}\left\{1+z^{-1}+z^{-2}\right\} \\ Y(z) &= \frac{1}{3}X(z)+\frac{1}{3}\frac{X(z)}{z}+\frac{1}{3}\frac{X(z)}{z^2} \end{aligned}$$

Taking the inverse z-transform, we have

$$y[n] = \frac{1}{3}x[n]+\frac{1}{3}x[n-1]+\frac{1}{3}x[n-2]$$

The block diagram representation of the filter in Direct-Form I is shown below:

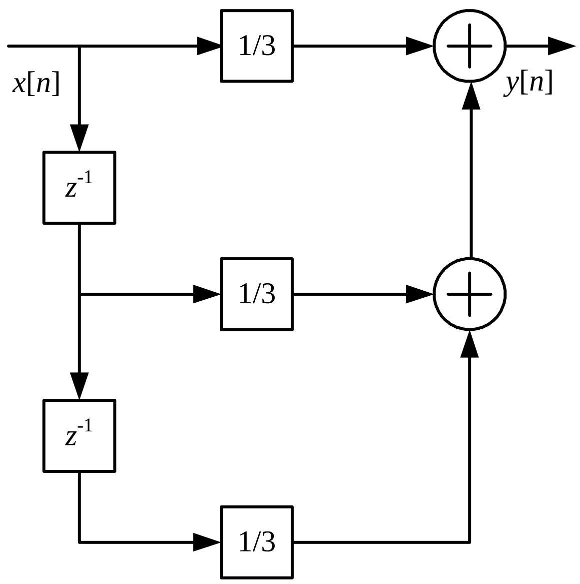

Figure 2: Direct-Form I implementation (FIR filter) for Question 1(a).

(b) Complete Solution

Given that

$$H(z) = \frac{2}{1-3z^{-1}+z^{-2}}$$

This is an IIR filter.

Re-writing the transfer function, we have

$$\begin{aligned}\frac{Y(z)}{X(z)} &= \frac{2}{1-3z^{-1}+z^{-2}} \\ Y(z) &= 2X(z)+3\frac{Y(z)}{z}-\frac{Y(z)}{z^2}\end{aligned}$$

Taking the inverse z-transform, we have

$$y[n] = 3y[n-1]-y[n-2]+2x[n]$$

The block diagram representation of the filter in Direct-Form I is shown below:

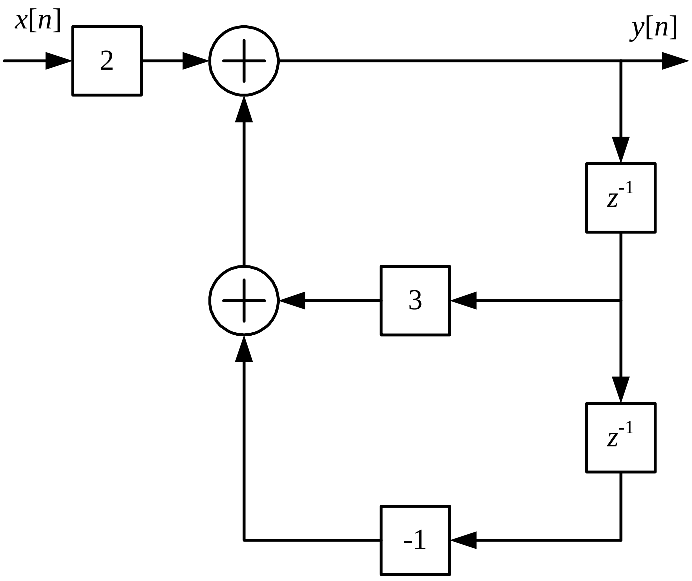

Figure 3: Direct-Form I implementation (IIR filter) for Question 1(b).

Q2

Partial Solution

The given transfer function is

$$H(z) = \frac{1}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$

The corresponding difference equation is

$$y[n] = x[n]-\frac{1}{4}y[n-1]+\frac{1}{8}y[n-2]$$

The block diagram representation of the filter in Direct-Form I is shown below:

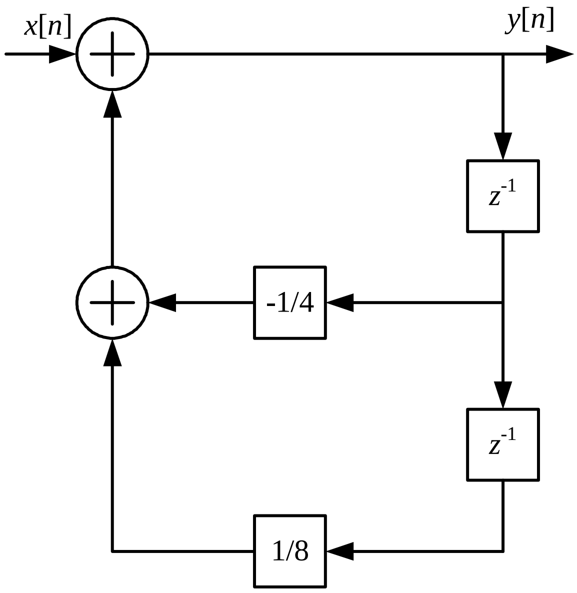

Figure 4: Direct-Form I implementation for Question 2.

Cascade Form

Factorising, the transfer function can be written as

$$H(z) = \frac{1}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} = H_1(z)H_2(z)$$

where

$$H_1(z) = \frac{1}{(1+\frac{1}{2}z^{-1})}$$

$$H_2(z) = \frac{1}{(1-\frac{1}{4}z^{-1})}$$

Using $H_1(z)$ and $H_2(z)$, the cascade form implementation of $H(z)$ can be drawn.

Parallel Form

Using partial fractions,

$$H(z) = \frac{2/3}{1+\frac{1}{2}z^{-1}}+\frac{1/3}{1-\frac{1}{4}z^{-1}} = H_3(z)+H_4(z)$$

Using $H_3(z)$ and $H_4(z)$, the parallel form implementation of $H(z)$ can be drawn.

The block diagram representation of the filter in Cascade Form is shown below:

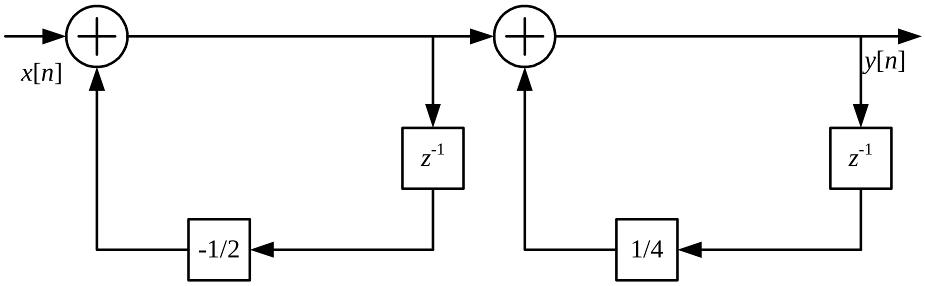

Figure 5: Cascade Form implementation for Question 2.

The block diagram representation of the filter in Parallel Form is shown below:

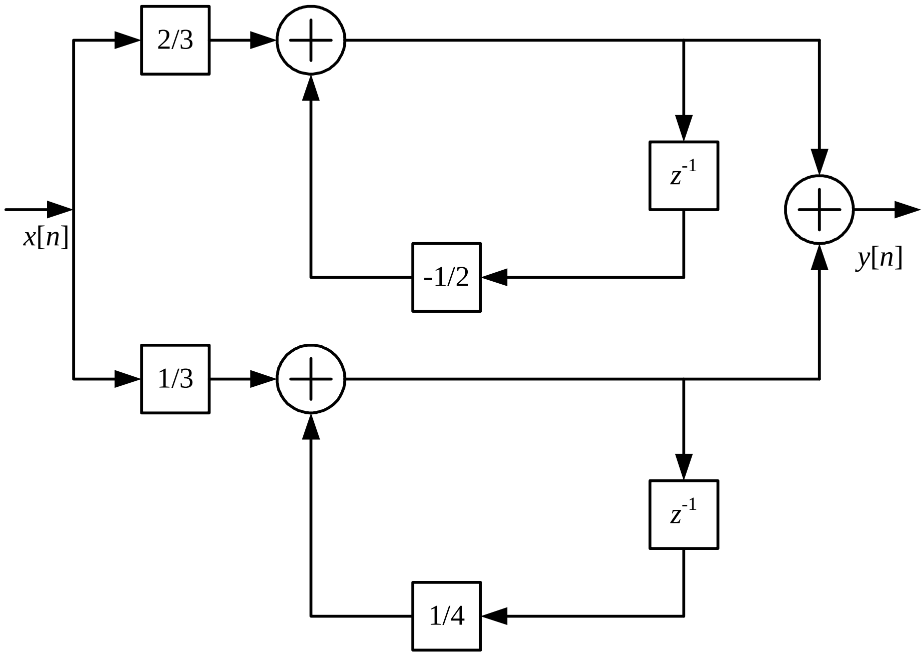

Figure 6: Parallel Form implementation for Question 2.

Q3

(a) Complete Solution

The given transfer function can be written as

$$H(z) = \frac{1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}} = H_1(z)\,H_2(z)$$

where

$$H_1(z) = \frac{1}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$
$$H_2(z) = 1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}$$

Implementation of $H_1(z)$

Re-writing $H_1(z)$, we have

$$\frac{Y_1(z)}{X_1(z)} = \frac{1}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$
$$Y_1(z) = X_1(z)-\frac{1}{4}\frac{Y_1(z)}{z}+\frac{1}{8}\frac{Y_1(z)}{z^2}$$

Taking the inverse z-transform, we have

$$y_1[n] = x_1[n]-\frac{1}{4}y_1[n-1]+\frac{1}{8}y_1[n-2]$$

The block diagram representation of $H_1(z)$ in Direct-Form I is shown below:

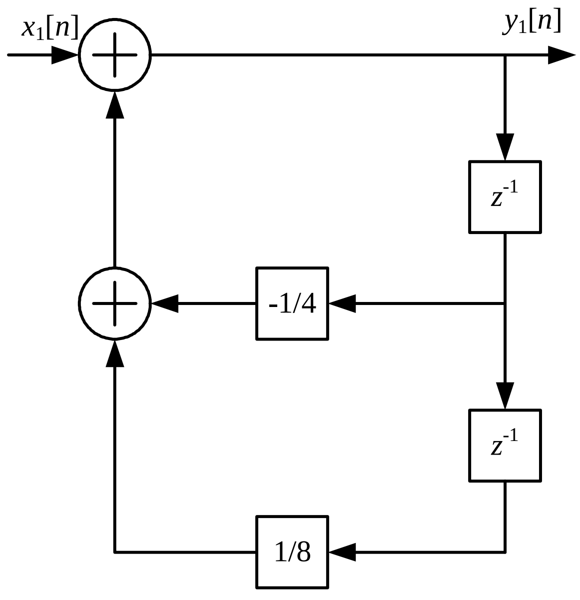

Figure 7: Direct-Form I implementation of $H_1(z)$ for Question 3(a).

Implementation of $H_2(z)$

Re-writing $H_2(z)$, we have

$$\frac{Y_2(z)}{X_2(z)} = 1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}$$
$$Y_2(z) = X_2(z)-\frac{7}{4}\frac{X_2(z)}{z}-\frac{1}{2}\frac{X_2(z)}{z^2}$$

Taking the inverse z-transform, we have

$$y_2[n] \quad = \quad x_2[n] - \frac{7}{4}x_2[n-1] - \frac{1}{2}x_2[n-2]$$

The block diagram representation of $H_2(z)$ in Direct-Form I is shown below:

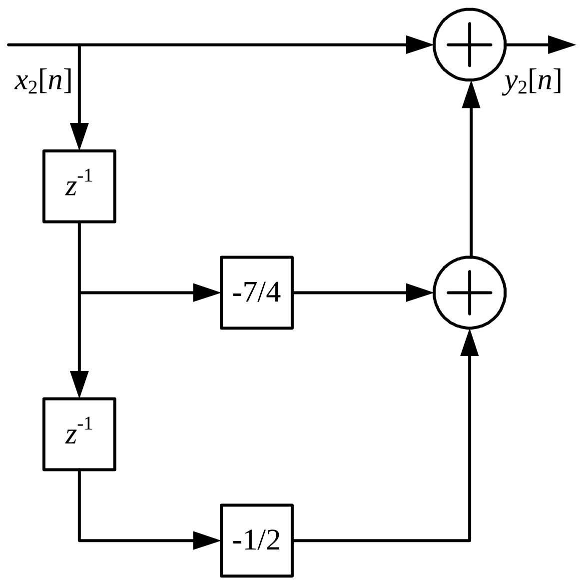

Figure 8: Direct-Form I implementation of $H_2(z)$ for Question 3(a).

Cascading the blocks, we have

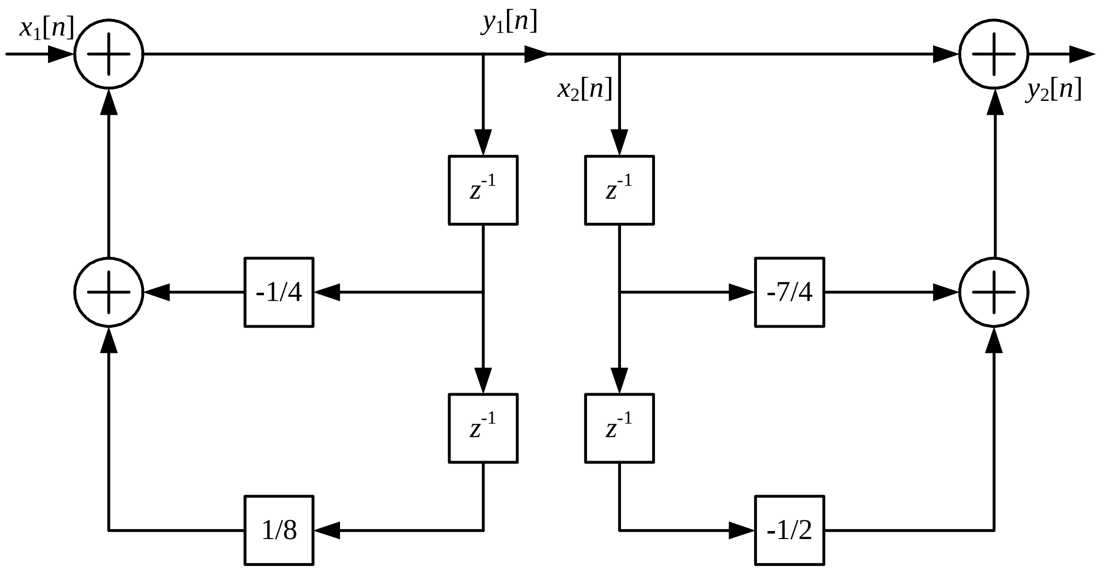

Figure 9: Direct-Form I implementation of $H(z)$ for Question 3(a).

Eliminating the common delay elements, we have

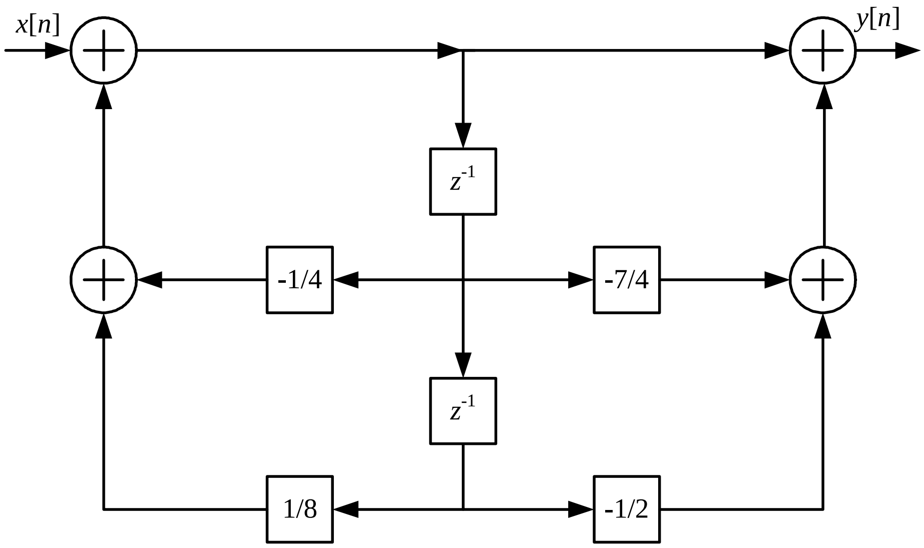

Figure 10: Direct-Form II implementation of $H(z)$ for Question 3(a).

(b) Solution

The difference equation representing the filter is

$$y[n] - \frac{2}{3}y[n-1] + \frac{1}{9}y[n-2] \quad = \quad x[n] - 6x[n-1] + 8x[n-2]$$

The block diagram representation of the filter in Direct-Form II is shown below:

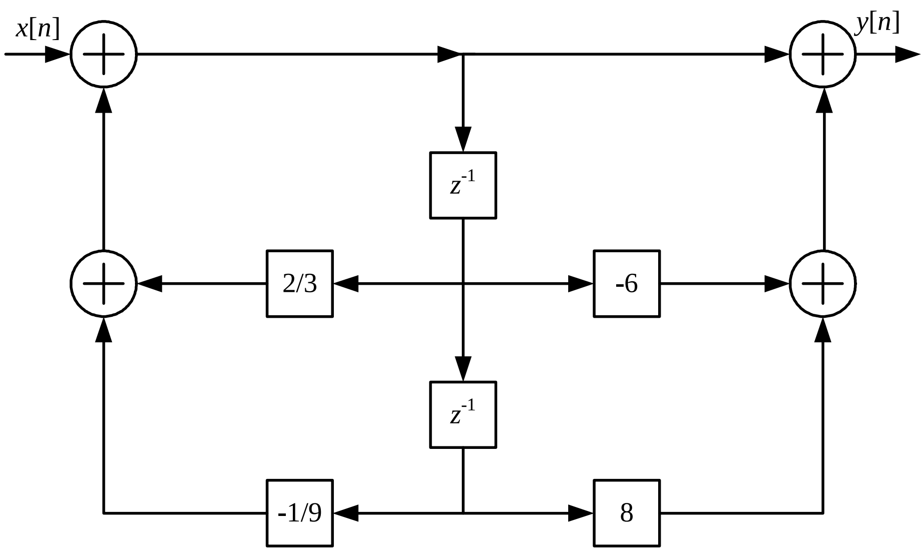

Figure 11: Direct-Form II implementation for Question 3(b).

Q4

Solution with Hint

The given block diagram is in Direct-Form II.
Show the following steps:

- Draw Direct-Form I implementation.
- Identify the cascaded blocks $H(z) = H_1(z)H_2(z)$.
- Find the transfer function for $H_1(z)$ and $H_2(z)$ respectively.
- Find the overall transfer function $H(z)$.
- From $H(z)$, find the difference equation.

The difference equation representing the filter is

$$y[n] + \frac{1}{3}y[n-1] - \frac{2}{9}y[n-2] \quad = \quad x[n] - \frac{9}{8}x[n-1]$$