AUSTRALIAN NATIONAL UNIVERSITY
Department of Engineering

ENGN6612/4612 Digital Signal Processing and Control
Problem Set #7 Fast Fourier Transform (FFT)

Q1

A discrete signal $x[n]$ is defined as:

(a)

$$x[n] = \begin{cases} 1 & \text{for } n = 4l+2 \\ 0 & \text{elsewhere} \end{cases}$$

with $l = 0, \pm 1, \pm 2, \cdots$.

(b)

$$x[n] = \begin{cases} 0 & \text{for } n = 0 \\ 1 & \text{for } n = 1,3 \\ 2 & \text{for } n = 2 \\ 0 & \text{elsewhere} \end{cases}$$

For each $x[n]$:

- State whether the signal is periodic, (non-periodic) finite or (non-periodic) finite duration.
- Calculate the 8-point DFT of $x[n]$.
- Assuming $x[n]$ is a finite duration signal (that exists only for $0 \leq n \leq 8$), calculate the DTFT of $x[n]$.
- Show that DFT is sampled version of DTFT (consider both real and imaginary parts).

Q2

Show that the FFT shown schematically in the figure below corresponds to a 4-point DFT. `(challenge problem)`

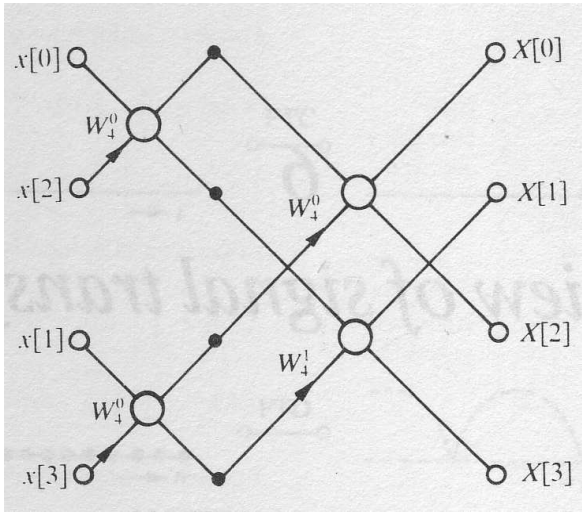

Figure 1: Question 2

Q3

A discrete signal $x[n]$ is defined as:
(a)

$$x[n] = \begin{cases} 1 & \text{for } n = 4l+2 \\ 0 & \text{elsewhere} \end{cases}$$

with $l = 0, \pm 1, \pm 2, \cdots$.

(b)

$$x[n] = \begin{cases} n+1 & \text{for } 0 \le n < 4 \\ 0 & \text{elsewhere} \end{cases}$$

For this signal:

- Calculate $X[k]$ using definition of DFT (take $N = 4$).
- Calculate $X[k]$ by making use of the diagram shown in Question 2.

Q4

Consider the periodic sequences $x_p[n]$ and $h_p[n]$ (with period $N = 4$):

(a)

$$h_p[n] = \begin{cases} n+1 & \text{for } 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$x_p[n] = \begin{cases} 1 & \text{for } n = 1, 2 \\ 0 & \text{elsewhere} \end{cases}$$

(b)

$$h_p[n] = \begin{cases} n & \text{for } 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$x_p[n] = \begin{cases} 1 & \text{for } n = 2 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the output $z_p[n] = x_p[n] \circledast h_p[n]$ using both (i) graphical discrete-time circular convolution and (ii) DFT method.