**AUSTRALIAN NATIONAL UNIVERSITY**
**Department of Engineering**

# ENGN6612/4612 Digital Signal Processing and Control
# Problem Set #6 Discrete Fourier Transform (DFT)

## Q1

The periodic function $x[n]$ is defined as:

(a)

$$x[n] = \begin{cases} 1 & \text{for } n = 4l + 2 \\ 0 & \text{elsewhere} \end{cases}$$

(b)

$$x[n] = \begin{cases} 1 & \text{for } n = 4l \text{ and } n = 4l + 3 \\ 2 & \text{for } n = 4l + 1 \text{ and } n = 4l + 2 \end{cases}$$

(c)`(challenge problem)`

$$x[n] = \begin{cases} 0 & \text{for } n = 4l \\ 1 & \text{for } n = 4l + 1 \text{ and } n = 4l + 3 \\ 2 & \text{for } n = 4l + 2 \end{cases}$$

with $l = 0, \pm 1, \pm 2, \cdots$.

For each $x[n]$:

- Plot the fundamental interval for $x[n]$.
- Calculate the $N$-point DFT of $x[n]$.
- Calculate and plot the magnitude and phase of DFT.
- Calculate and plot the real and imaginary parts of DFT..

## Q2

The $N$-point DFT $X[k]$ is defined as:

(a) $N = 4$

$$X[k] = \begin{cases} 1 & \text{for } k = 4l \text{ and } k = 4l + 1 \text{ and } k = 4l + 2 \text{ and } k = 4l + 3 \\ 0 & \text{elsewhere} \end{cases}$$

(b) $N = 4$

$$X[k] = \begin{cases} 2 & \text{for } k = 4l + 1 \\ 2 & \text{for } k = 4l + 3 \\ 0 & \text{elsewhere} \end{cases}$$

(c) $N = 16$ `(challenge problem)`

$$X[k] = \begin{cases} 2 & \text{for } k = 16l + 1 \text{ and } k = 16l + 15 \\ 1 & \text{for } k = 16l + 3 \text{ and } k = 16l + 13 \\ 0 & \text{elsewhere} \end{cases}$$

with $l = 0, \pm 1, \pm 2, \cdots$.

For each $X[k]$:

- Plot the fundamental interval for $X[k]$.
- Calculate the $N$-point IDFT of $X[k]$.
- Plot the fundamental interval for $x[n]$.

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# ENGN6612/4612 Digital Signal Processing and Control
# Problem Set #6 Solution

## Q1

### (a) Complete Solution

Given that

$$x[n] = \begin{cases} 1 & \text{for } n = 4l+2 \\ 0 & \text{elsewhere} \end{cases}$$

For fundamental interval $l = 0$. Hence

$$x[n] = \begin{cases} 1 & \text{for } n = 2 \\ 0 & \text{for } n = 0,1,3 \end{cases}$$

The plot of fundamental interval of $x[n]$ is shown in the figure below:

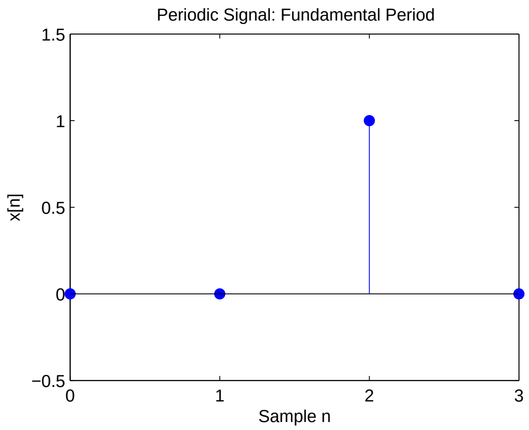



Figure 1: Question 1(a)

The 4-point DFT of $x[n]$ is

$$\begin{aligned} X[k] &= \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn} \\ &= \sum_{n=0}^{3} x[n]e^{-j\frac{\pi}{2}kn} \\ &= x[2]e^{-j\pi k} \\ &= e^{-j\pi k} \end{aligned}$$

Hence

$$\begin{aligned} X[0] &= e^{-j0} = 1 = 1\angle 0^\circ \\ X[1] &= e^{-j\pi} = -1 = 1\angle 180^\circ \\ X[2] &= e^{-j2\pi} = 1 = 1\angle 0^\circ \\ X[3] &= e^{-j3\pi} = -1 = 1\angle 180^\circ \end{aligned}$$

The plot of magnitude $|X[k]|$ and phase $\angle X[k]$ is shown in the figures below:

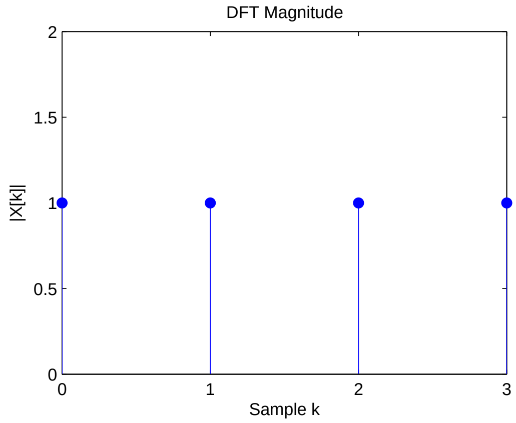



Figure 2: Question 1(a): Magnitude of DFT

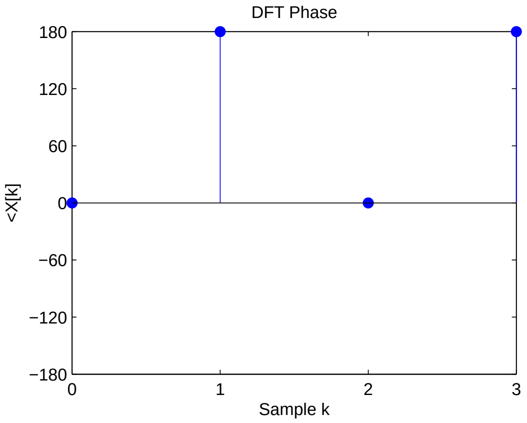



Figure 3: Question 1(a): Phase of DFT

The plot of real part $\Re\{X[k]\}$ and imaginary part $\Im\{X[k]\}$ is shown in the figures below:

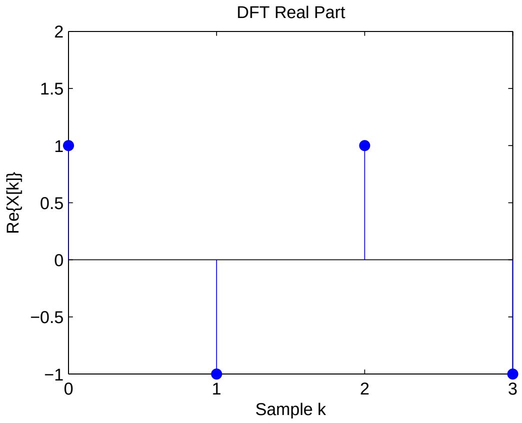



Figure 4: Question 1(a): Real part of DFT

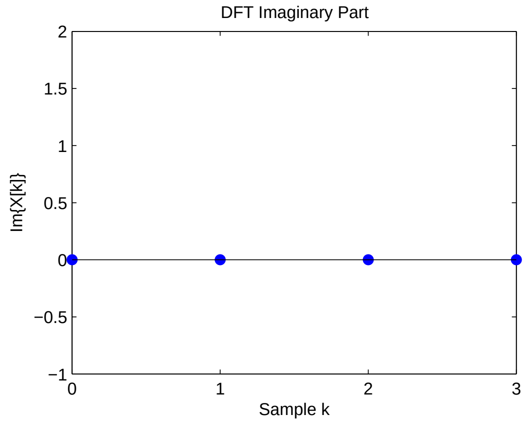


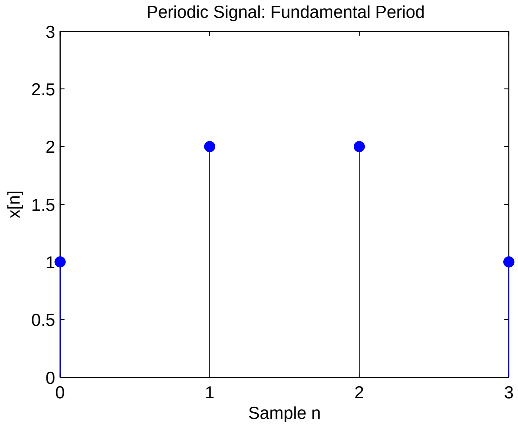

Figure 5: Question 1(a): Imaginary part of DFT

Check answer in Matlab using the following commands

```
>> n=[0 1 2 3];
>> x=[0 0 1 0];
>> X=fft(x);
>> MagX=abs(X);
>> PhaseX=angle(X)*180/pi;
>> RealX=real(X);
>> ImagX=imag(X)
```

## (b) Solution

The plot of fundamental interval of $x[n]$ is shown in the figure below:




The plot of magnitude $|X[k]|$, phase $\angle X[k]$, real part $\Re\{X[k]\}$ and imaginary part $\Im\{X[k]\}$are:

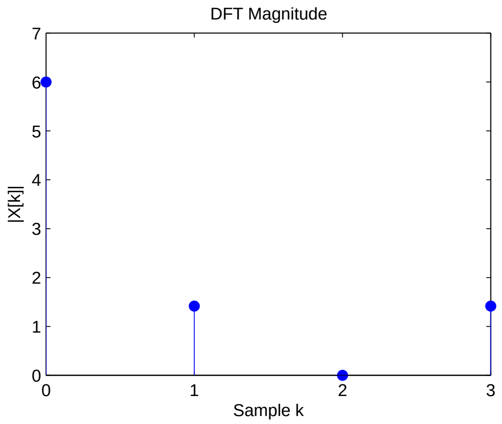



Figure 6: Question 1(b): Magnitude of DFT

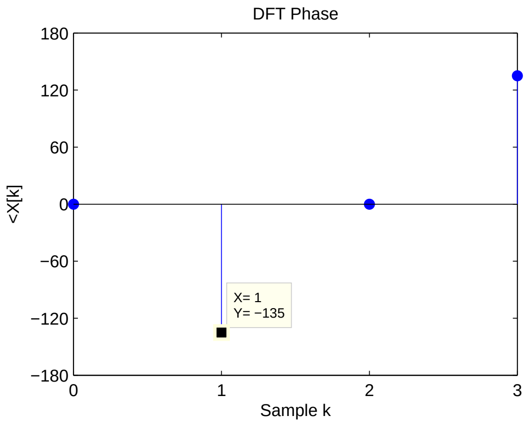



Figure 7: Question 1(b): Phase of DFT

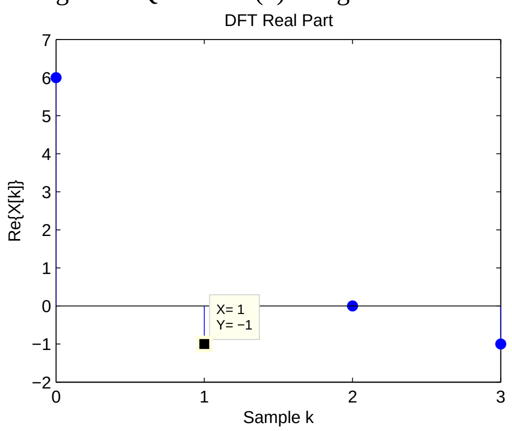



Figure 8: Question 1(b): Real part of DFT

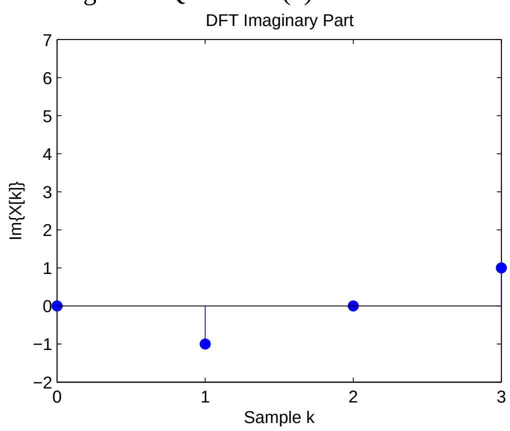
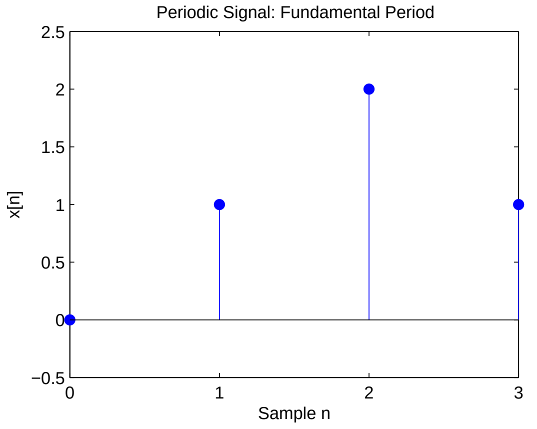



Figure 9: Question 1(b): Imaginary part of DFT

## (c) Solution

The plot of fundamental interval of $x[n]$ is shown in the figure below:




The plot of magnitude $|X[k]|$, phase $\angle X[k]$, real part $\Re\{X[k]\}$ and imaginary part $\Im\{X[k]\}$are:

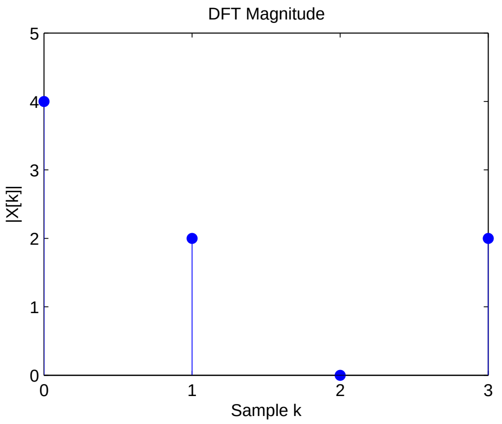



Figure 10: Question 1(c): Magnitude of DFT

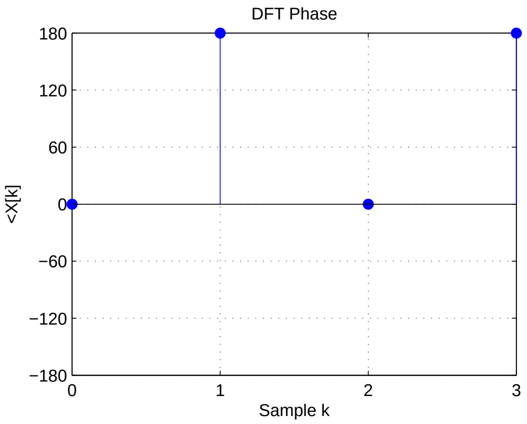



Figure 11: Question 1(c): Phase of DFT

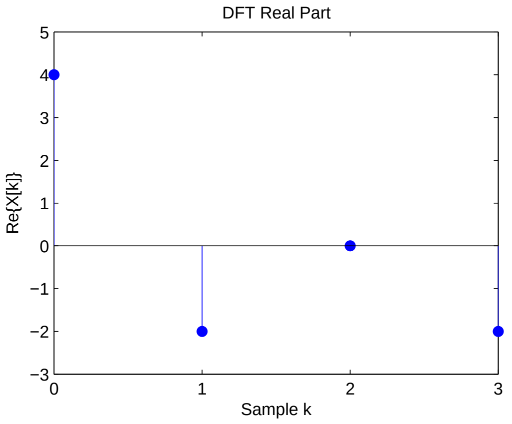



Figure 12: Question 1(c): Real part of DFT

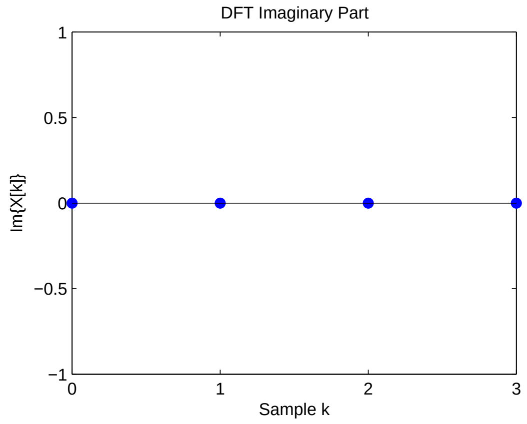



Figure 13: Question 1(c): Imaginary part of DFT

# Q2

## (a) Complete Solution

Given that

$$X[k] = \begin{cases} 1 & \text{for } k = 4l \text{ and } k = 4l+1 \text{ and } k = 4l+2 \text{ and } k = 4l+3 \\ 0 & \text{elsewhere} \end{cases}$$

For fundamental interval $l = 0$. Hence

$$X[k] = 1 \text{ for } k = 0, 1, 2, 3$$

The 4-point IDFT of $X[k]$ is

$$\begin{aligned} x[n] &= \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn} \\ &= \frac{1}{4}\sum_{k=0}^{3} x[n]e^{j\frac{\pi}{2}kn} \\ &= \frac{1}{4}\left\{x[0]e^{j0} + x[1]e^{j\frac{\pi}{2}n} + x[2]e^{j\pi n} + x[3]e^{j\frac{3\pi}{2}n}\right\} \\ &= \frac{1}{4}\left\{1 + e^{j\frac{\pi}{2}n} + e^{j\pi n} + e^{j\frac{3\pi}{2}n}\right\} \end{aligned}$$

Hence

$$\begin{aligned} x[0] &= \frac{1}{4}\left\{1 + e^{j\frac{\pi}{2}0} + e^{j\pi 0} + e^{j\frac{3\pi}{2}0}\right\} = \frac{1}{4}(1+1+1+1) = 1 \\ x[1] &= \frac{1}{4}\left\{1 + e^{j\frac{\pi}{2}} + e^{j\pi} + e^{j\frac{3\pi}{2}}\right\} = \frac{1}{4}(1+j-1-j) = 0 \\ x[2] &= \frac{1}{4}\left\{1 + e^{j\pi} + e^{j2\pi} + e^{j3\pi}\right\} = \frac{1}{4}(1-1+1-1) = 0 \\ x[3] &= \frac{1}{4}\left\{1 + e^{j\frac{3\pi}{2}n} + e^{j3\pi} + e^{j\frac{9\pi}{2}n}\right\} = \frac{1}{4}(1-j-1+j) = 0 \end{aligned}$$

The plot of fundamental interval of $X[k]$ and $x[n]$ is shown in the figures below:

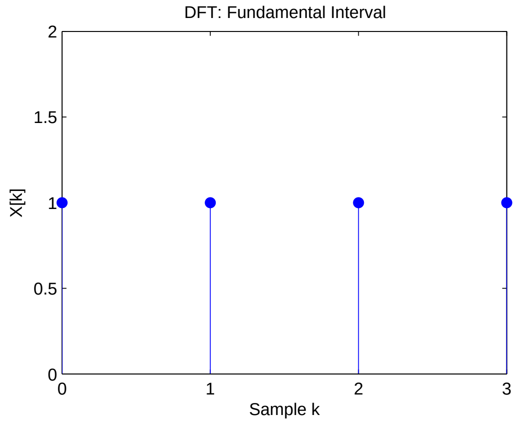



Figure 14: Question 2(a): DFT Fundamental Interval

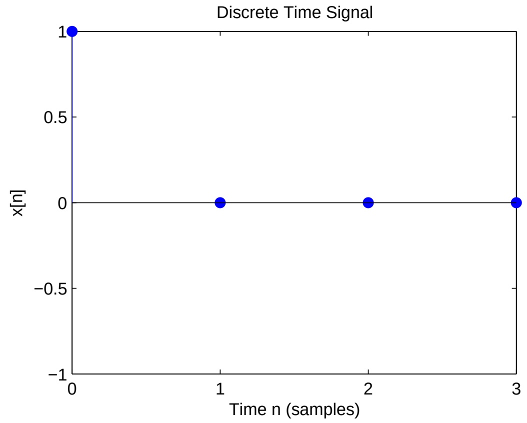



Figure 15: Question 2(a): Periodic signal

Check answer in Matlab using the following commands

```
>> k=[0 1 2 3];
>> X=[1 1 1 1];
>> x=ifft(X);
```

# Q2

## (b) Partial Solution

Given that

$$X[k] = \begin{cases} 2 & \text{for } k = 4l+1 \\ 2 & \text{for } k = 4l+3 \\ 0 & \text{elsewhere} \end{cases}$$

For fundamental interval $l = 0$. Hence

$$X[k] = \begin{cases} 2 & \text{for } k = 1,3 \\ 0 & \text{for } k = 0,2 \end{cases}$$

The 4-point IDFT of $X[k]$ is

$$\begin{aligned} x[n] &= \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn} \\ &= \frac{1}{4}\sum_{k=0}^{3} x[n]e^{j\frac{\pi}{2}kn} \\ &= \frac{1}{4}\left\{x[1]e^{j\frac{\pi}{2}n} + x[3]e^{j\frac{3\pi}{2}n}\right\} \\ &= \frac{1}{2}\left\{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right\} \\ &= \cos\left(\frac{n\pi}{2}\right) \\ &= \cos\left(\frac{2n\pi}{4}\right) \end{aligned}$$

The plot of fundamental interval of $X[k]$ and $x[n]$ is shown in the figures below:

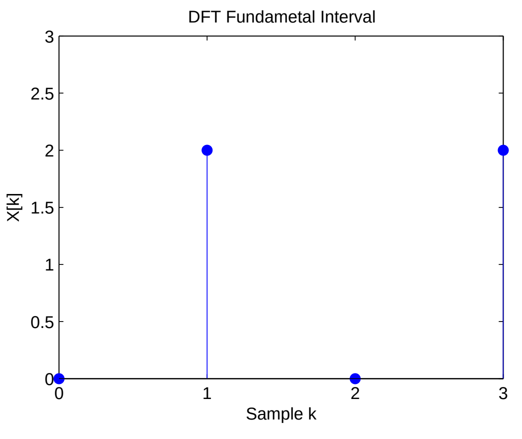



Figure 16: Question 2(b): DFT Fundamental Interval

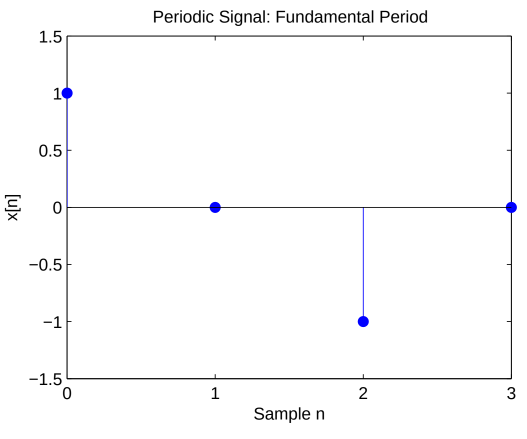



Figure 17: Question 2(b): Periodic signal

# Q3

## (c) Solution

Show that 4-point IDFT of $X[k]$ is

$$x[n] = \frac{1}{4}\cos(\pi n/8) + \frac{1}{8}\cos(3\pi n/8)$$

The plot of fundamental interval of $X[k]$ and $x[n]$ is shown in the figures below:

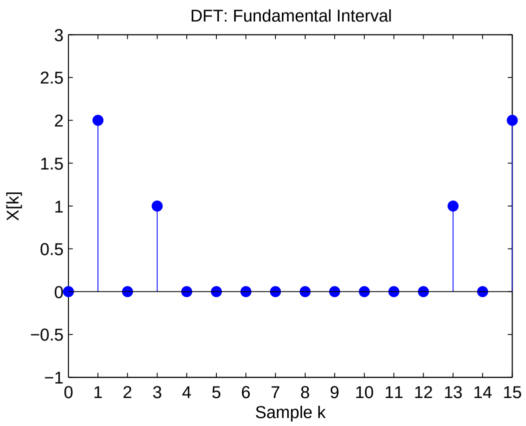



Figure 18: Question 2(c): DFT Fundamental Interval

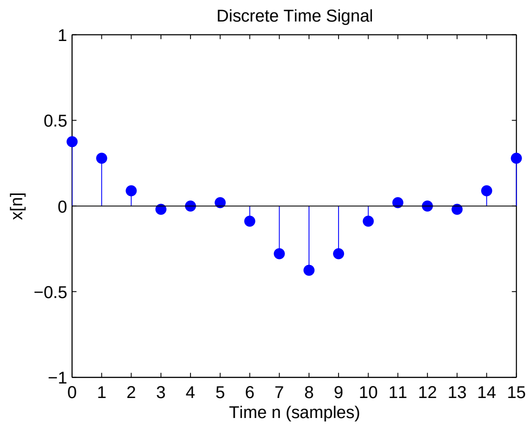



Figure 19: Question 2(c): Periodic signal

Check answer in Matlab using `ifft` command.
See also `L10_DFT.m`