THE AUSTRALIAN NATIONAL UNIVERSITY

Sample Practice Exam Questions 2005

ENGN4612/ENGN6612

Digital Signal Processing and Control

Study Period: 15 minutes
Writing Period: 180 minutes

Permitted materials:
(i) Non-programmable calculator
(ii) A4 page (one sheet) with hand-written notes on both sides.

Instructions:
Answer all five questions.
Write your answers for the DSP Section (Q1,Q2,Q3) in script book A.
Write your answers for the Control Section (Q4,Q5) in script book B.
Hand in the exam question sheets as well as the two script books.

Each question is worth 20 marks. Parts of the question carry the number of marks indicated. You must explain and show all steps taken to arrive at your answer. The clarity and precision of your explanations and answers will be taken into account when marking. All plots/sketches must be appropriately labelled. The MATLAB *commands must be written one command per line following the* MATLAB *syntax. You must always indicate the units of all physical quantities. Some useful formulas are provided at the end of this paper.*

DSP SECTION

Question 1: [20 marks total]

(a) Consider the digital filter defined by the difference equation:

$$y[n]-0.5y[n-1]=x[n]+x[n-1]$$

(i) Find the transfer function $H(z)$ and the impulse response $h[n]$. [**2 marks**]

(ii) Explain whether the filter FIR or IIR. [**1 marks**]

(iii) Find explicitly (analytically) the frequency response magnitude $|H(e^{j\omega})|$ and phase $\angle H(e^{j\omega})$. [**4 marks**]

(iv) Draw the direct form II implementation of the filter. [**3 marks**]

(b) Consider a discrete-time LTI system with the input $x[n]$ and impulse response $h[n]$ given below:

$$\begin{aligned} x[n] &= \delta[n]+2\delta[n-1] \\ h[n] &= 0.5\delta[n]-2\delta[n-1]+\delta[n-2] \end{aligned}$$

Determine the output $y[n]$ using discrete-time graphical convolution. [**5 marks**]

(c) **(i)** Consider the impulse response $h[n]$ of a digital filter shown in the Figure 1.

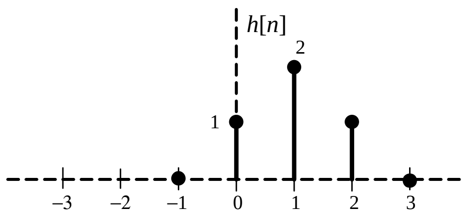

Figure 1: Impulse response of digital filter (Question 1(d)

Write the set of MATLAB commands (typically three lines expected) to *plot* the Bode plot for the given filter. [**3 marks**]

(ii) Explain in words what is the meaning of *time-invariance* property of digital systems. [**2 marks**]

CONTROL SECTION

Question 4: [20 marks total]

The discrete state space representation for an inverted pendulum without feedback is given by

$$\begin{aligned} x[n+1] &= Ax[n] + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u[n] \\ y[n] &= \begin{pmatrix} 0 & 1 \end{pmatrix} x[n] \end{aligned}$$

(a) Assume that the control u is scalar. How many elements does the state vector x have? How many elements does the measurement y have? [**2 marks**]

(b) Write down the explicit form of A in the state equation above, given the sampling time is $T_s = 0.02$ seconds, and comment on the stability of the inverted pendulum without and with feedback. [**4 marks**]

(c) Is this system reachable? [**2 marks**]

(d) Find numerical values for the state-feedback controller gains k, to stabilize the inverted pendulum. [**4 marks**]

(e) Is this system observable? [**2 marks**]

(f) Explain the consequences of observability and non-observability. [**2 marks**]

(g) What does the separation principle enable you to achieve in controlling the inverted pendulum? Illustrate how the feedback process works for this system. [**4 marks**]

USEFUL FORMULAS

DSP:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$1 + 1 = 2$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$$

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}, \qquad \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn}$$

LQR:

$$X = Q + K'RK + (A-BK)'X(A-BK)$$

$$K = [R + B'XB]^{-1}B'XA$$

$$u_n = -Kx_n$$

Kalman filter:

$$\hat{x} = m_x + \frac{\sigma_x^2}{\sigma_x^2 + \sigma_v^2}(y - m_x)$$

$$\sigma^2 = \sigma_x^2 - \frac{\sigma_x^2\sigma_x^2}{\sigma_x^2 + \sigma_v^2}$$

$$\hat{x}_{n+1|n} = A\hat{x}_{n|n-1} + Bu_n + L(y_n - C\hat{x}_{n|n-1})$$

$$Y = A[Y - YC'[CYC' + Rn]^{-1}CY]A' + GQnG'$$

$$L = AYC'[CYC' + Rn]^{-1}$$

$$\begin{aligned} \hat{x}_{n+1|n} &= A\hat{x}_{n|n-1} + Bu_n + L_n(y_n - C\hat{x}_{n|n-1}) \\ L_n &= AY_{n|n-1}C'[CY_{n|n-1}C' + R]^{-1} \\ Y_{n+1|n} &= A[Y_{n|n-1} - Y_{n|n-1}C'[CY_{n|n-1}C' + R]^{-1}CY_{n|n-1}]A' + GQG' \end{aligned}$$

$$\begin{aligned} \hat{x}_{n|n} &= \hat{x}_{n|n-1} + Y_{n|n-1}C'[CY_{n|n-1}C' + R]^{-1}(y_n - C\hat{x}_{n|n-1}) \\ Y_{n|n} &= Y_{n|n-1} - Y_{n|n-1}C'[CY_{n|n-1}C' + R]^{-1}CY_{n|n-1} \end{aligned}$$