ENGN6612 & ENGN4612
Digital Signal Processing & Control
Computer Labs
Eigenpractice
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1 Theory

We would like to find numbers λ and nonzero vectors v such that

$$Av = \lambda v \tag{1}$$

We can rewrite this as

$$(\lambda I - A)v = 0 \tag{2}$$

If equation (2) holds for some $v \neq 0$, then we must have

$$\det(\lambda I - A) = 0 \tag{3}$$

This is an equation in λ alone, which we can solve for λ. Once values of λ have been found, i.e. the eigenvalues, we use them in (2) and solve for the corresponding eigenvectors v.

Note: if

$$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

then

$$\det(B) = ad - bc$$

2 Example

Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

Then

$$\lambda I - A = \begin{pmatrix} \lambda - 1 & 0 \\ 0 & \lambda - 2 \end{pmatrix}$$

and

$$\det(\lambda I - A) = (\lambda - 1)(\lambda - 2) = 0$$

gives

$$\lambda = 1 \text{ or } \lambda = 2$$

For $\lambda = 1$, solve (2) for v:

$$0 = (1I - A)v = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This gives $b = 0$ and a is free. Therefore the eigenvectors corresponding to the eigenvalue $\lambda = 1$ have the form

$$v_1 = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

where a is a free parameter. For definiteness, we can take

$$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

since this vector has unit length and spans the subspace of eigenvectors for $\lambda = 1$.

Similarly, for $\lambda = 2$ we obtain

$$v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

3 Practice

Find the eigenvalues and eigenvectors for the following matrices:

1. $$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

2. $$A = \begin{pmatrix} 1 & -1 \\ 3 & 1 \end{pmatrix}$$