ENGN6612 & ENGN4612
Digital Signal Processing & Control
Computer Labs
CLAB3
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1 Aim

1. Practice DTFT and DFT.
2. Digital Filters and FFT.
3. To revise and/or acquire and develop skills in using MATLAB simulations.

2 DTFT

QUESTION 2.1 *(DTFT and time shifts.)*

1. *Consider the infinite extent signal $x[n]$ given by*

n	$\cdots$	-1	0	1	2	3	$\cdots$
$x[n]$	$\cdots$	0	1	2	3	0	$\cdots$

That is, $x[0] = 1$, etc. Find the DTFT $X(e^{j\omega})$.

2. *Find the time shifted signals*

n	$\cdots$	-1	0	1	2	3	4	5	$\cdots$
$x[n]$	$\cdots$	0	1	2	3	0	0	0	$\cdots$
$x[n-1]$	?	?	?	?	?	?	?	?	?
$x[n-2]$	?	?	?	?	?	?	?	?	?
$x[n-3]$	?	?	?	?	?	?	?	?	?

3. *Find directly the DTFT of each time shifted signal.*
4. *Use these results to verify the time shift property of the DTFT (time shift $\leftrightarrow$ multiplication by an exponential).*

QUESTION 2.2 *(DTFT and (linear) convolution.)*

1. *Consider the infinite extent signals $x[n]$ and $y[n]$ given by*

n	$\cdots$	-1	0	1	2	3	$\cdots$
$x[n]$	$\cdots$	0	1	2	3	0	$\cdots$
$y[n]$	$\cdots$	0	2	1	4	0	$\cdots$

Find the DTFT $Y(e^{j\omega})$ of $y[n]$.

2. *Find the (linear) convolution $x[n] * y[n]$:*

n	$\cdots$	-1	0	1	2	3	4	5	$\cdots$
$x[n] * y[n]$	?	?	?	?	?	?	?	?	?

3. *Use your results to verify the convolution property of the DTFT (convolution $\leftrightarrow$ multiplication).*

3 DFT

QUESTION 3.1 *(DFT and circular time shifts.)*

1. *Consider the finite extent signal* $x[n]$ *of length* $N = 3$ *given by*

n	0	1	2
$x[n]$	1	2	3

   Find the DFT $X[k]$.

2. *Check that* $X[k]$ *can be obtained by sampling* $X(e^{j\omega})$ *obtained in Question 2.1.*

3. *Find the circular time shifted signals*

n	0	1	2
$x[n]$	1	2	3
$x[n-1]$	?	?	?
$x[n-2]$	?	?	?
$x[n-3]$	?	?	?

4. *Find directly the DFT of each time shifted signal.*

5. *Use these results to verify the circular time shift property of the DFT (circular time shift* $\leftrightarrow$ *multiplication by an exponential).*

QUESTION 3.2 *(DFT and circular convolution.)*

1. *Consider the finite extent signals* $x[n]$ *and* $y[n]$ *both of length* $N = 3$ *given by*

n	0	1	2
$x[n]$	1	2	3
$y[n]$	2	1	4

   Find the DFT $Y[k]$ *of* $y[n]$.

2. *Find the circular convolution* $x[n] \otimes y[n]$:

n	0	1	2
$x[n] \otimes y[n]$	?	?	?

3. *Use your results to verify the circular convolution property of the DFT (convolution* $\leftrightarrow$ *multiplication).*

4 Digital Filters

QUESTION 4.1 *Consider the digital filter defined by the difference equation*

$$y[n] + 0.5y[n-1] = u[n] - u[n-1]$$

1. *Find the transfer function* $H(z)$.

2. *Is the filter stable?*

3. *Find the impulse response* $h[n]$.

4. *Is the filter FIR or IIR?*
5. *Find explicitly (analytically) the frequency response magnitude $|H(e^{j\omega})|$ and phase $\angle H(e^{j\omega})$.*
6. *Draw the direct form I implementation.*
7. *Draw the direct form II implementation.*
8. *Write down equations for a state space realisation of the filter.*
9. *Is the realisation reachable?*
10. *Is the realisation observable?*

MATLAB EXERCISE 4.2

In MATLAB, you can specify a transfer function

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}} \tag{1}$$

with sample time $T_s = 1$ as follows:

```
Ts=1;
num=[b0 b1 b2 b3];
den=[a0 a1 a2 a3];
H=tf(num,den,Ts,'variable','z^-1');
```

The transfer function object H can then be used for a variety of purposes, such as for producing a *Bode diagram*

```
bode(H);
```

or *root locus*

```
rlocus(H);
```

The objects a and b are in general of the appropriate lengths, depending on the degrees of the corresponding polynomial in z.

Also, if needed, a sampling time can be specified by using a value different from 1, say 0.01, etc. In MATLAB, a sampling time $T_s = -1$ indicates that the sampling time has not been specified.

1. Use MATLAB to plot the Bode diagram of the filter you worked on in Question 4.1.
2. Compare with the analytical expression you obtained for the frequency response.
3. What happens when you try the command fvtool(H). What is one other limitation of fvtool (this is related to Problem Set 05 Q2 part c).

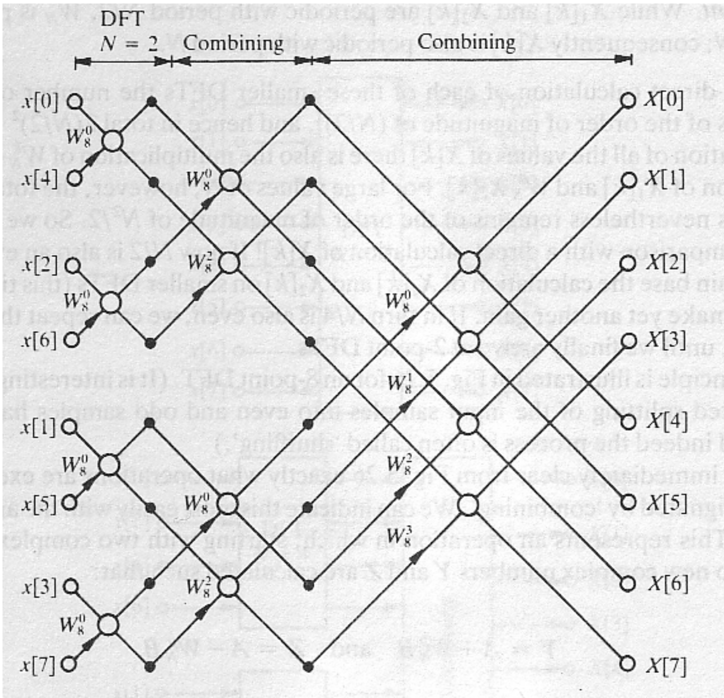

Figure 1: Flow-graph of 8-point FFT with butterfly computation.

5 FFT

QUESTION 5.1 *1. Consider the flow-graph of 8-point radix-2 decimation-in-time FFT algorithm shown in Fig. 1.*

2. *In the flow graph, write the equations for the intermediate points in the two combining stages.*
3. *Find the expressions for DFT samples $X[k]$ in terms of the input samples $x[n]$.*
4. *Use the results derived above to find the 8-point DFT of the finite signal given by*

n	0	1	2	3	4	5	6	7
$x[n]$	1	2	3	4	0	0	0	0

5. *Use Matlab command* `fft` *to check your answer.*

6 Miscellaneous

QUESTION 6.1 *1. Let $N = 2$ and consider the two DFT basis signals (see lecture notes)*

$$b_0 = \left((b_0)_0,\ (b_0)_1 \right)$$
$$b_1 = \left((b_1)_0,\ (b_1)_1 \right)$$

Find numerical values for these signals and show that

$$(b_n, b_m) = \begin{cases} 2 & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases}$$

for $n, m = 0, 1$.

2. *Find the Toeplitz matrices for the FIR and IIR filters in the notes.*
3. *Explain why FIR filters are stable.*
4. *Show that the direct form I filter given in the notes is an implementation of the given difference equation (or transfer function).*
5. *Repeat Question 4.1 and* MATLAB *Exercise 4.2 for the filter*

$$y[n] + 0.25y[n-1] - 0.125y[n-2] = u[n]$$