ENGN6612 & ENGN4612
Digital Signal Processing & Control
Computer Labs
CLAB2
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1 Aim

1. To review some fundamental concepts about signals including sampling and Fourier transforms.

2. To revise and/or acquire and develop skills in using Matlab simulations.

3. Practice FT and ZT.

2 DFT and Windows

In CLAB1 we effectively used a *rectangular* window applied to the signal before applying the `fft` algorithm (DFT). In order to better resolve spectra of signals, a range of window shapes have been proposed. The *Hamming* window is one of the most important, and is defined by

$$\text{ham}_n = 0.54 - 0.46\cos(2\pi n/(N-1))$$

Use of the Hamming window results in improved spectral resolution. We now investigate this using Matlab.

Matlab Exercise 2.1
`hamming1.m`

1. Create a folder in your directory for CLAB2.

2. Start Matlab and set the working directory to your CLAB2 folder.

3. Download the file hamming1.m and save it to your working Matlab folder.

4. Run the file by typing

```
>> hamming1
```

   You will need to hit return to step through the script. At each stage pay close attention to what is being done. In particular, for both the rectangular and Hamming window cases, note

   (a) the shape of the window in the time and frequency domains[1]

   (b) the effect of the window on the signal in both domains

   Summarise the main points you observed.

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[1] The `fftshift` command allows use to plot the spectrum in the baseband in the range $[-\omega_s/2, \omega_s/2]$ instead of the default $[0, \omega_s]$.

5. `Did use of the Hamming window improve the resolution of the spectrum? Explain in terms of what you see.`

6. `Open the file hamming1.m in your m-file editor. Look for the window width parameter` p `near the top of the file. Re-do the simulations for a range of values of` p `between` 0 `and` 0.4.

7. `Comment on the influence of window width on spectrum resolution.`

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3 Sampling

In this section we take a look at sampling using MATLAB. Before getting into this, let's do some thought experiments concerning continuous time filtering.

QUESTION 3.1 *Consider a pair of continuous time signals of frequencies 25 Hz and 75 Hz:*

$$\begin{aligned} x_1(t) &= \sin(2\pi 25t) \\ x_2(t) &= \sin(2\pi 75t) \end{aligned}$$

Now imagine passing these signals through an ideal low pass filter (LPF) H with a cutoff frequency of 50 Hz:

$$\begin{aligned} y_1(t) &= Hx_1(t) \\ y_2(t) &= Hx_2(t) \end{aligned}$$

Sketch the graphs of the signals in the time domain and the magnitude spectra in the frequency domain for all four signals.

Keep these results in the back of your minds to help with what follows.

To prepare for our work on sampling we will first create a digital LPF with a cut-off frequency of 50 Hz using daily sampling frequency of $f_s = 100$ Hz using MATLAB's `fdatool` (filter design tool). The `fdatool` integrates the `fvtool` (filter visualization tool), discussed in Lecture 06.

MATLAB EXERCISE 3.2

1. `Open the filter design tool by typing`

```
>> fdatool
```

2. `In the fdatool window, select the following parameters:`

```
filter type    lowpass
design method  IIR Chebyshev Type 1
filter order   minimum
freq units     Hz
Fs             100
Fpass          49
Fstop          49.5
mag units      dB
Apass          1
Astop          80
```

`Click the Design Filter button, and then look at the displayed frequency response (check that it is the desired LPF).`

3. Export the filter to your MATLAB workspace in terms of coefficients: File -> Export... You should see data items SOS and G in your workspace. These will be used below.

We next sample the signals $x_1(t)$ and $x_2(t)$ using the sampling rate of 100 Hz and examine the results in the time and frequency domains.

MATLAB EXERCISE 3.3
sampling1.m

1. Create samples of the two signals $x_1(t)$ and $x_2(t)$ using the sampling rate of 100 Hz as follows:

```
fs=100;
ts=1/fs;
ws=2*pi*fs;
t = 0:ts:10;
x1 = sin(2*pi*25*t);
x2 = sin(2*pi*75*t);
figure;
subplot(2,1,1)
plot(t,x1); title('f=25 hz, fs=100 Hz'); axis([0 0.5 -1 1]);
subplot(2,1,2)
plot(t,x2); title('f=75 hz, fs=100 Hz'); axis([0 0.5 -1 1]);
```

   Hmm, what's going on?

2. Now apply the digital LPF you created above to both signals:

```
y1=sosfilt(SOS,x1);
y2=sosfilt(SOS,x2);
figure;
subplot(2,1,1)
plot(t,y1); title('f=25 hz, fs=100 Hz, LPF at 50 Hz'); axis([0
0.5 -1 1]);
subplot(2,1,2)
plot(t,y2); title('f=75 hz, fs=100 Hz, LPF at 50 Hz'); axis([0
0.5 -1 1]);
```

   Again, what's going on?

3. Obtain the magnitude spectra for all four signals.

4. Download and save in your working directory the file sampling1.m. This script attempts to create a broadband signal of bandwidth f Hz (this parameter appears near the top of the file), again for $f_s = 100$ Hz. Set $f = 25$ Hz, and run the script. Repeat for $f = 75$ Hz. Describe what you see.

5. Explain what you are observing in the above investigations. Compare with the ideal LPF results in continuous time you obtained in Question 3.1.

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4 Questions and Exercises

QUESTION 4.1   *1. Consider the transfer function*

$$H(z) = \frac{1 - \frac{1}{3}z^{-1}}{(1 - z^{-1})(1 + 2z^{-1})}$$

(a) Find the inverse z-transform $h[n]$ of $H(z)$ using partial fractions, z-transform properties, and the table of z-transform pairs.

(b) Plot $h[n]$ for $0 < n < 4$. Use initial value theorem to confirm value of $x[0]$.

(c) Find the difference equation corresponding to $H(z)$.

2. Consider a discrete time Linear Time Invariant (LTI) system described by the difference equation

$$y[n-1] + 2y[n] = x[n]$$

(a) Determine the transfer function $H(z)$ and the impulse response $h[n]$. Is the system causal or non-causal?

(b) Determine the pole-zero plot for the system. Is the system stable or unstable?

(c) Determine the output $y[n]$ if the input signal is $x[n] = (\frac{1}{4})^n u[n]$.

(d) For $0 < n < 4$, plot the input $x[n]$, impulse response $h[n]$ and calculated output $y[n]$.

3. Find the impulse response $h(t)$ of the ideal continuous time LPF discussed in the notes (see page 21, section 9 on Sampling).

4. Verify the expression for the Fourier series of the pulse train $s(t)$ given in the notes (see section 9 on Sampling).